

Nontopological self-dual Maxwell-Higgs vortices

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We study the existence of self-dual nontopological vortices in generalized Maxwell-Higgs models recently introduced in Ref. [1]. Our investigation is explicitly illustrated by choosing a sixth-order self-interaction potential, which is the simplest one allowing the existence of nontopological structures. We specify some Maxwell-Higgs models yielding BPS nontopological vortices having energy proportional to the magnetic flux, Φ_B , and whose profiles are numerically achieved. Particularly, we investigate the way the new solutions approach the boundary values, from which we verify their nontopological behavior. Finally, we depict the profiles numerically found, highlighting the main features they present.

I. INTRODUCTION

This work deals with the presence of nontopological vortices in generalized Maxwell-Higgs models recently introduced in Ref. [1], whose dynamics is controlled by two positive functions depending on the Higgs field only. Before focusing on this, however, it seems interesting to first comment on topological solutions. As one knows, topologically nontrivial configurations, generically named topological defects, are frequently described as static solutions to some nonlinear classical field models allowing for the spontaneous symmetry breaking mechanism [2]. In this sense, such structures have been studied intensively all over the years, their consequences being applied to many areas of interest, specially in Cosmology, since they are known to be formed during symmetry breaking phase transitions [3].

The interesting point to be raised is that, according to the usual approach, these solutions can be found by solving a given set of first-order differential equations (instead of the second-order Euler-Lagrange ones) [4]. The main advantage of following such a prescription is that it also reveals that the resulting solutions are stable against decaying into the respective mesons (i.e. they saturate a lower bound for the energy of the model), with their energy depending only on the physical fields boundary conditions (i.e. the topology). The simplest solution is the static kink coming from a (1+1)-dimensional model described by one real scalar field only [5]. In addition, there are also vortices and magnetic monopoles: the first ones are born in (1+2)-dimensional gauged Abelian-Higgs theory [6], while the second ones stand for the static solutions appearing in (1+3)-dimensional gauged non-Abelian-Higgs models [7].

Recently, many authors have also investigated topological configurations coming from generalized models possessing nonusual dynamics. In these works interesting new properties have been discovered, including variations on the shape of the resulting solitons, see Refs. [8, 9]. In addition, some of these models have been verified to support first-order solutions and a lower bound for their

energy, see Ref. [10]. Here, it is important to point out that the main motivation regarding noncanonical dynamics comes in a rather natural way from the string theory, and that interesting applications of such an idea have been found in connection with the accelerated inflationary phase of the universe [11], tachyon matter [12], dark matter [13], and others [14].

In field theory models, finite energy solutions require that the self-interacting potential goes to zero as the field solutions approach their asymptotic profiles. Stable nontopological solutions satisfy asymptotical boundary conditions that imply finite energy and null topological charge (becoming stable due to conservation of Noether charges). Nontopological defects with vorticity $n > 0$ have null profiles at origin and far from it, requiring a ϕ^6 potential, as it occurs in the Chern-Simons-Higgs model [15] and in some extended electrodynamics [16]. Nontopological solutions were initially studied in models constituted of one and two scalar fields, with applications in QCD [17, 18], see also Ref. [19]. Some investigations about the conditions under which modified Lagrangians, composed of generalized kinetic terms, yield nontopological soliton solutions were undertaken in Ref. [20].

The impossibility to attain nontopological solutions in a Maxwell-Higgs model is associated to the usual ϕ^4 self-interacting potential, which provides BPS solutions. A generalized Maxwell-Higgs model with dielectric functions depending on the scalar field, was recently proposed [1], and may be used to achieve nontopological solutions. In the present work, we investigate new aspects of the Maxwell-Higgs model of Ref. [1], showing that it also supports nontopological configurations whenever supplemented with self-interacting potentials that engender symmetric minimum.

In order to present our results, this work is organized as follows. In Sec. II, we review the way the adopted nonusual Abelian-Higgs model engenders self-duality, embracing the first-order equations and the lower bound for the overall energy (defined in terms of an auxiliary function conveniently introduced). In Sec. III, we define the scenarios we consider by choosing the Higgs potential suitably. In addition, we characterize our so-

lutions by investigating the way they approximate the boundary values, which the physical fields are supposed to reach (near the origin and asymptotically). We also show that, in the appropriate limit, these models admit the very same analytical solutions and related conclusions. We go further by presenting numerical profiles, which explicitly describe the general properties of the solutions. We still point out the existence of models engendering entirely analytical nontopological solutions (i.e. not approximate ones). Finally, in Section IV, we present our concluding remarks and perspectives regarding future investigations.

II. THE MODEL

We begin by reviewing the first-order framework firstly introduced in Ref. [1] for attaining topological solutions, whose starting-point is the (1 + 2)-dimensional generalized Maxwell-Higgs Lagrangian density,

$$\mathcal{L} = -\frac{G(|\phi|)}{4}F_{\mu\nu}F^{\mu\nu} + w(|\phi|)|D_\mu\phi|^2 - U(|\phi|), \quad (1)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength tensor, $D_\mu\phi = \partial_\mu\phi + ieA_\mu\phi$ is the minimal covariant derivative. Here, we use standard conventions, including the plus-minus signature for the Minkowski metric. The positive functions, $G(|\phi|)$ and $w(|\phi|)$, generalize the dynamics of the usual Maxwell-Higgs model. It is easy to show that the Gauss law of this model is saturated by the temporal gauge, $A_0 = 0$, so it describes only magnetic configurations, as the usual case.

Our aim is looking for radially symmetric configurations coming from (1). For such purpose we implement the standard stationary vortex Ansatz,

$$\phi(r, \theta) = v g(r) e^{in\theta}, \quad \mathbf{A}(r, \theta) = -\frac{\hat{\theta}}{er}(a(r) - n), \quad (2)$$

where $n = \pm 1, \pm 2, \pm 3, \dots$ stands for the winding number of the topological configuration.

We focus our attention on the self-dual configurations of Lagrangian (1), governed by two coupled first-order equations that minimize the total energy of the model. In order to obtain such equations, we follow the usual Bogomol'nyi-Prasad-Sommerfield (BPS) formalism by writing the energy density of this model in the radially symmetric Ansatz,

$$\varepsilon = \frac{G}{2e^2} \left(\frac{1}{r} \frac{da}{dr} \right)^2 + v^2 w \left(\left(\frac{dg}{dr} \right)^2 + \frac{a^2 g^2}{r^2} \right) + U, \quad (3)$$

where $U = U(g)$. In the present scenario, self-duality only holds when $G(|\phi|)$, $w(|\phi|)$ and $U(|\phi|)$ are constrained to each other by

$$\frac{d}{dg} \sqrt{GU} = -\sqrt{2}ev^2wg. \quad (4)$$

Under such a constraint, the energy density Eq. (3) can be rewritten as,

$$\varepsilon = \frac{G}{2} \left(\frac{1}{er} \frac{da}{dr} \pm \sqrt{\frac{2U}{G}} \right)^2 + v^2 w \left(\frac{dg}{dr} \mp \frac{ag}{r} \right)^2 \mp \frac{1}{er} \frac{d}{dr} (a\sqrt{2GU}), \quad (5)$$

from which one concludes that the corresponding total energy saturates its lower bound when $g(r)$ and $a(r)$ satisfy

$$\frac{dg}{dr} = \pm \frac{ag}{r}, \quad (6)$$

$$\frac{1}{er} \frac{da}{dr} = \mp \sqrt{\frac{2U}{G}}. \quad (7)$$

These are the BPS or self-dual first order equations of the generalized model. Saturating the first-order equations, the energy density is shown to be

$$\varepsilon_{bps} = \mp \frac{1}{er} \frac{dH}{dr}, \quad (8)$$

where we have defined the auxiliary function

$$H(r) = a\sqrt{2GU}. \quad (9)$$

By using BPS equations, we also can write the correspondent energy density as

$$\varepsilon_{bps} = 2U + 2v^2 w \left(\frac{ag}{r} \right)^2, \quad (10)$$

which will be positive definite for $w(g) > 0$. In summary, given G , w and U constrained by relation (4), the first-order equations (6) and (7) give rise to self-dual solutions for both $g(r)$ and $a(r)$ which satisfying the boundary conditions,

$$g(0) = 0, \quad g(\infty) = 1, \quad (11)$$

$$a(0) = n, \quad a(\infty) = 0, \quad (12)$$

engendering topological configurations. These solutions possess finite energy obtained by integrating Eq. (8) over the plane, i.e.

$$E_{bps} = 2\pi \int r \varepsilon_{bps} dr = \pm 2\pi [H(0) - H(\infty)], \quad (13)$$

with $H(0)$ and $H(\infty) = 0$ being the values of $H(r)$ at the boundaries. In this model, suitable choices of functions G and w allowed to achieve finite energy BPS solutions associated with potentials U different from the usual ϕ^4 one [1], with the energy resulting proportional to the magnetic flux. So far, no study about the possibility of attaining BPS nontopological solutions in this Maxwell-Higgs framework was developed.

III. NONTOPOLOGICAL VORTICES

In this Section, we show that the generalized Maxwell-Higgs model of Ref. [1] may also support nontopological radially symmetric solitons, assuming some specific choices for the generalized functions G and w , and that the first-order equations (6) and (7) are solved fulfilling suitable boundary conditions. We can show that the energy of these BPS nontopological magnetic vortices is proportional to the magnetic flux Φ_B , not necessarily quantized in this case. From now on and for simplicity, we choose $e = v = 1$, and consider only the case $n > 0$, corresponding to the upper signs in Eqs. (6), (7), (8), (13).

In order to obtain nontopological configurations, the profile functions $g(r)$ and $a(r)$ must obey the following boundary conditions for $n \neq 0$,

$$g(0) = 0, \quad g(\infty) = 0, \quad (14)$$

$$a(0) = n, \quad a(\infty) = -\alpha_n, \quad (15)$$

where α_n stands for a positive real number calculated numerically. Under such boundary conditions, the magnetic flux Φ_B is immediately obtained

$$\Phi_B = 2\pi \int r B(r) dr = 2\pi(n + \alpha_n), \quad (16)$$

and is not necessarily quantized (i.e. α_n is not an integer).

Notwithstanding the magnetic flux be given by (16), it is still possible to define classical field models for which the magnetic flux defines the lower bound for the BPS energy (13). Note that this occurs only when the function H satisfies boundary conditions proportional to the ones of Eq. (15), which requires a particular choice for the functions G and w .

It is well-known that nontopological solutions arise in field theories, which possess at least one symmetric vacuum, and asymptotical conditions compatible with finite energy. In canonical field models, the simplest Higgs-potential consistent with this requirement is the usual sixth-order one,

$$U(g) = \frac{\lambda^2}{2} g^2 (1 - g^2)^2, \quad (17)$$

where λ stands for the coupling constant for the scalar-matter self-interaction (supposed to be dimensionless and positive). In this case, the symmetric vacuum is $g = 0$, the asymmetric one standing for $g = 1$. Here, we point out that the simplest self-dual Maxwell-Higgs model (saturated by the usual $|\phi|^4$ -potential) presents only the asymmetric vacuum $g = 1$ and does not support nontopological structures. Within our generalized scenario, we circumvent this problem by adopting the $|\phi|^6$ -potential (17), whilst choosing the functions G and w conveniently.

Without loss of generality, and in the context of Lagrangian (1), we suppose that the function $G(g)$ has the

following behavior at boundaries, i.e. when $g(r) \rightarrow 0$,

$$G(g) = \frac{\gamma}{g^2} + \gamma_0 + \gamma_2 g^2 \dots, \quad (18)$$

with the constants $\gamma, \gamma_0, \gamma_2, \dots$ characterizing such behavior. The case $\gamma \neq 0$ allows to express the total BPS energy as being proportional to the magnetic flux. In the following we fix $\gamma = 1$.

By using the Eqs. (17) and (18), we should investigate the way the fields $g(r)$ and $a(r)$ behave near the boundaries. We thus solve the first-order equations (6) and (7) around the boundary values (14) and (15), obtaining the behavior of profiles near the origin,

$$g(r) \approx C_0 r^n, \quad (19)$$

$$a(r) \approx n - \frac{\lambda C_0^2}{2(n+1)} r^{2n+2}, \quad (20)$$

and asymptotically,

$$g(r) \approx \frac{C_\infty}{r^{\alpha_n}}, \quad (21)$$

$$a(r) \approx -\alpha_n + \frac{\lambda C_\infty^2}{2(\alpha_n - 1)} \frac{1}{r^{2\alpha_n - 2}}. \quad (22)$$

Here, C_0 and C_∞ are positive real constants obtained numerically by requiring proper behavior near and far from the origin, respectively.

The point to be raised here is that the solutions (19) and (20) stand for a typical behavior of a topological vortex near the origin whereas Eqs. (21) and (22) encode the typical nontopological profile far from the origin. It is worthwhile to say that in Ref. [15], it was shown that the very same dependence in r (despite some numerical factors) characterizes the nontopological vortices of the usual Chern-Simons-Higgs model.

For small g , replacing expressions (17) and (18) in the self-dual Eqs. (6) and (7), one achieves a Liouville's equation for $g(r)$ (in accordance with the procedure of Ref. [15]), whose analytical solution is

$$g(r) = \frac{2(n+1)}{r_0 \sqrt{\lambda}} \frac{\left(\frac{r}{r_0}\right)^n}{\left(\frac{r}{r_0}\right)^{2(n+1)} + 1}, \quad (23)$$

with its maximum located at $r = r_0 (n/(n+2))^{1/(2n+2)}$ (for n sufficiently large, its location matches r_0). The corresponding expression for the gauge function $a(r)$ reads as

$$a(r) = -n - 2 + \frac{2(n+1)}{\left(\frac{r}{r_0}\right)^{2(n+1)} + 1}. \quad (24)$$

The (approximate) solutions attained this way fulfill conditions (14) and (15), and hold for $r \rightarrow 0$ or $r \rightarrow \infty$, with $\alpha_n = n + 2$, confirming the nontopological behavior.

IV. SOME MODELS ENDOWED WITH NONTOPOLOGICAL SOLUTIONS

We now show some models belonging to the generalized Maxwell-Higgs Lagrangian (1), endowed with the sixth-order potential (17). These models are better characterized by choosing a specific function $w(g)$, from which the constraint (4) yields $G(g)$ behaving as (18).

The first model is defined by the potential (17) and the functions

$$w(g) = \frac{2}{3}\lambda(g^2 + 1), \quad G(g) = \frac{(g^2 + 3)^2}{9g^2}. \quad (25)$$

From Eqs. (6) and (7), we get the self-dual equations of this model,

$$\frac{dg}{dr} = \frac{ag}{r}, \quad \frac{1}{r} \frac{da}{dr} = \frac{3\lambda g^2 (g^2 - 1)}{g^2 + 3}, \quad (26)$$

which was firstly studied in Ref. [1] (except for some numerical factors) for analysis of topological solitons. The BPS nontopological configurations of this specific model satisfy the boundary conditions (14) and (15), with energy given by Eq. (13). From the choices of this model, we calculate $H(0) = n\lambda$ and $H(\infty) = -\alpha_n\lambda$, so that the total energy is

$$E_{bps} = 2\pi\lambda(n + \alpha_n). \quad (27)$$

thus we verify that the energy of the resulting self-dual structures is proportional to their magnetic flux.

The second $|\phi|^6$ -model we study is defined by the functions

$$w(g) = \lambda, \quad G(g) = \frac{1}{g^2}, \quad (28)$$

whilst the self-dual equations are

$$\frac{dg}{dr} = \frac{ag}{r}, \quad \frac{1}{r} \frac{da}{dr} = \lambda g^2 (g^2 - 1), \quad (29)$$

whose topological solutions were analyzed in Ref. [1]. We observe that Eqs. (29) mimic those of the standard self-dual Chern-Simons-Higgs model [15] for $\lambda = 2/\kappa^2$ (κ standing for the Chern-Simons constant). Despite satisfying the very same differential equations, the solutions we describe in this work are physically different, since they present no electric charge.

By solving the BPS equations (29) around the boundary values (14) and (15), we verify that $g(r)$ and $a(r)$ are given as in Eqs. (19) and (20) near the origin, while their asymptotic behavior coincides with the ones of (21) and (22). Thus, the solutions coming from (29) indeed stand for nontopological self-dual vortices possessing no electric charge. The total BPS energy is computed by using Eqs. (17), (28), (14) and (15), leading to $H(0) = n\lambda$, $H(\infty) = -\alpha_n\lambda$, and $E_{bps} = 2\pi\lambda(n + \alpha_n) = \lambda\Phi_B$, being proportional to the magnetic flux again.

At this point, it is interesting to note that because of Eqs. (26) and (29) the values of the winding number n are limited by the requirement $g \leq 1$, otherwise we do not obtain solutions satisfying the boundary conditions or with finite energy.

The third $|\phi|^6$ -model to be scrutinized is one that affords equations, which have an analytical solution. It is specified by the functions

$$w = 2\lambda(1 - g^2), \quad G(g) = \frac{(1 - g^2)^2}{g^2}, \quad (30)$$

providing the following self-dual equations:

$$\frac{dg}{dr} = \frac{ag}{r}, \quad \frac{1}{r} \frac{da}{dr} = -\lambda g^2. \quad (31)$$

The Eqs. (31) are exactly solvable, having analytical solutions given by (23) and (24), which hold for all r , not only at origin or at infinity. These solutions fulfill conditions (14) and (15), with $\alpha_n = n + 2$, possessing magnetic flux $\Phi_B = 4\pi(n + 1)$ and total energy $E_{bps} = 4\pi\lambda(n + 1)$.

The requirement of a positive-definite energy density (10) or positive $w(g)$ in (30) is equivalent to demanding $g \leq 1$, which holds only when

$$\lambda r_0^2 > n^{n/(n+1)} (n + 2)^{(n+2)/(n+1)} \quad (32)$$

is satisfied. Therefore, for a given value of the coupling constant λ , the possible values of the vorticity are limited in such a way that larger values of r_0 imply larger values of n .

We now show the numerical solutions for fixed $\lambda = 100$ and $r_0 = 1$. For these values, the winding number is restricted to $1 \leq n \leq 4$, $1 \leq n \leq 5$, or $1 \leq n \leq 8$, for the first, the second and the third models, respectively. It is worthwhile to note that a similar situation holds in the Chern-Simons-Higgs model [15], whose the BPS equations are the very same as those of the second model, see (29).

In Figs. 1 and 2, we depict the solutions for $g(r)$ and $a(r)$, respectively. The first ones are peaked at rings around the origin, whereas the second ones stand for peaks centered at the origin. It is interesting to note that, for the first model one finds $\alpha_1 = 3.03795$, $\alpha_2 = 4.07752$, and $\alpha_1 = 3.02977$, $\alpha_2 = 4.06176$, for the second model. Therefore, we find a small deviation of the value $\alpha_n = n + 2$.

The Figs. 3 and 4 show the solutions for the magnetic field $B(r) = -(da/dr)/r$ and for the self-dual energy density (8), respectively. In general, all the profiles exhibit the same behavior, being ring-like peaks centered at the origin, and vanishing asymptotically.

V. ENDING COMMENTS

In this work, we have investigated the way the generalized Maxwell-Higgs model (1) gives rise to self-dual

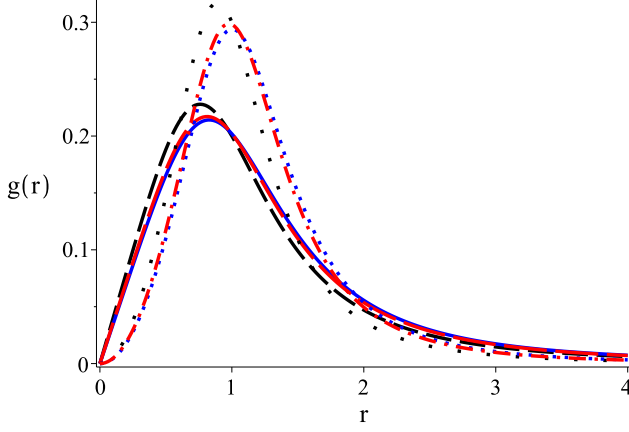


FIG. 1: Solutions to $g(r)$ calculated via (i) (26) (solid blue line for $n = 1$, dotted blue line for $n = 2$), (ii) (29) (long-dashed red line for $n = 1$, dash-dotted red line for $n = 2$), and (iii) (31) (i.e., the analytical profiles (23) and (24); dashed black line for $n = 1$, space-dotted black line for $n = 2$). We have used $\lambda = 100$ and $r_0 = 1$.

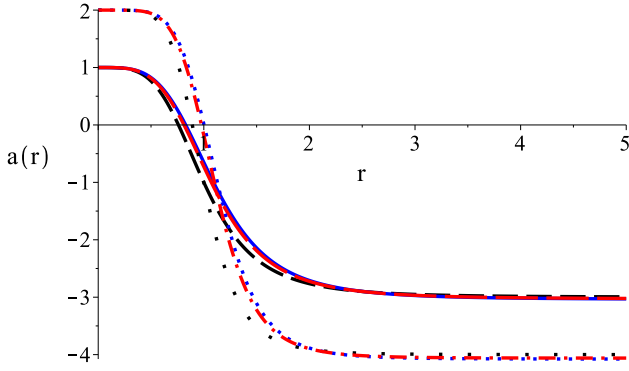


FIG. 2: Solutions to $a(r)$. Conventions as in Fig. 1.

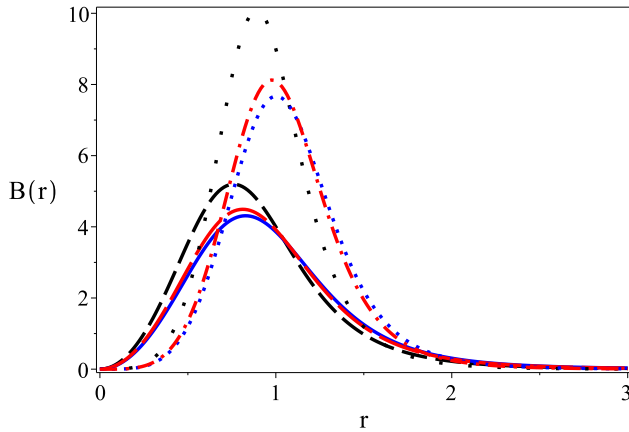


FIG. 3: Solutions to $B(r)$. Conventions as in Fig. 1.

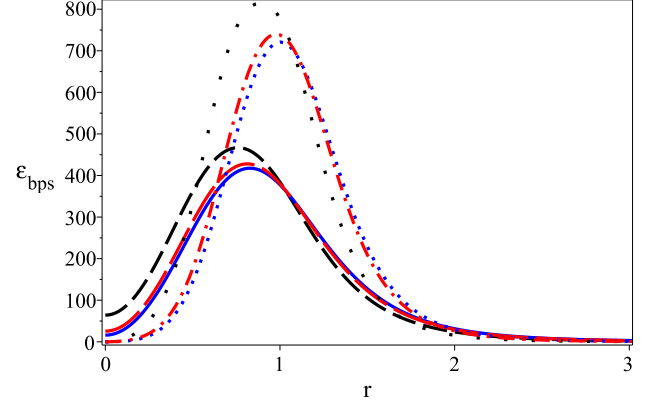


FIG. 4: Solutions to ε_{bps} . Conventions as in Fig. 1.

nontopological magnetic vortices possessing finite energy and no electric charge. In order to present our results, we have first reviewed the basic features of the model, which engenders self-duality only when a particular constraint (4) is satisfied. We have specified our Maxwell-Higgs models by choosing the usual sixth-order potential (17) and changing the function $w(|\phi|)$ conveniently, from which we have obtained the corresponding expression for $G(|\phi|)$ from the constraint (4). Without loss of generality, we have normalized the function $G(g)$ in such way that the energy of the nontopological solutions is proportional to the corresponding magnetic flux.

The BPS equations of the models here analyzed when $g \rightarrow 0$ reduce to exactly the equations (31). In this field regimen the solutions are exact and given by Eqs. (23) and (24). These equations and their solutions are invariant under the following scaling transformation,

$$r \rightarrow \delta r, \quad g(\delta r) = \delta^{-1} g(r), \quad a(\delta r) = a(r), \quad (33)$$

with δ the transformation parameter. Such an invariance enhances the solutions of the models because there must exist a free parameter defining a family of infinite solutions. In $|\phi|^6$ -models defined in $(1+2)$ -dimensions the λ coupling constant is dimensionless. Thus it becomes a free parameter characterizing the solutions. For example, λ determines the behavior of the solutions of (31) given by Eqs. (23) and (24) when $g \rightarrow 0$. This way, when $r \rightarrow 0$:

$$g(r) \simeq \frac{2(n+1)}{r_0 \sqrt{\lambda}} \left(\frac{r}{r_0} \right)^n + \dots, \quad (34)$$

and for $r \rightarrow \infty$:

$$g(r) \simeq \frac{2(n+1)}{r_0 \sqrt{\lambda}} \left(\frac{r_0}{r} \right)^{n+2} + \dots \quad (35)$$

An important result is that for fixed λ and r_0 , we can determine the values of n for which exist nontopological BPS vortices, as given by Eq. (32). In fact, we have verified that the winding number values are limited by

the condition $g \leq 1$. Despite this inequality (32) is exact only for the solution of the set of BPS equations (31) and it gives an upper-bound for the values of n in all models we have considered.

An other result is related to the BPS energy of the nontopological vortices; for the models we have studied, the energy depends explicitly on the free parameter λ , see (27).

The stability issue of the new nontopological BPS solutions is under investigation and the results will be reported elsewhere.

An other interesting issue concerns the search for BPS solutions in generalized Born-Infeld-Higgs models. This issue is now under consideration, and we expect interesting results for a future contribution.

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